A surprising optical property of Plexiglas rods—An unusual approach to birefringence

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Plexiglas (Lucite) is optically anisotropic, it scatters laser light, it has a high transparency for light, and it is economical and easy to cut and polish. Combining these properties, a lot of interesting applications are possible, which are very useful for teaching optics. The demonstration of elliptically polarized light in a direct and impressive way or the easy construction of quarter-wave or half-wave retardation plates with exceptional performance are a few examples.

I. INTRODUCTION

Ordinary Plexiglas rods,[1] which are normally used as light guides in laboratories, are optically anisotropic. This property is a consequence of the fabrication process. The rods are obtained in an indirect way: cast-sheet Plexiglas is cut into rectangular bars of desired thickness and then the bars are turned and polished. The sheets, from which the bars are cut, are fabricated by polymerizing methyl-metacrylate that takes place in cuvettes with glass walls. The polymerization process, however, is accomplished by a volume contraction. Since the lid of the cuvette is free to move, this contraction is only possible perpendicular to the sheet surface. This preferred contraction direction induces a small linear birefringence (constant, negative uniaxial) always with the optic axis perpendicular to the rod axis.

Furthermore, local inhomogeneities of the refractive index induce Rayleigh scattering of light in the rod. But, this scattering process is only visible when the light energy in the Plexiglas is confined to a rather small volume or cylinder as in a laser beam. Then, in a completely darkened room, even a large audience will observe the path of the laser beam in the rod (see Fig. 1). The distribution of the scattered light (Rayleigh scattering) is the same as in the case of an electric dipole. Therefore, the local state of polarization can be derived from the scattered light distribution in a plane normal to the propagation direction of the laser beam (a dipole does not emit in the direction of its axis). Combining these two properties of Plexiglas it is possible to demonstrate the influence of a birefringent material on linearly polarized light in a direct and impressive way.[2]

II. DEMONSTRATION OF ELLIPTICAL POLARIZED LIGHT

The birefringence of Plexiglas can be demonstrated as in Fig. 1. In addition the optic axis of the rod is rotated 45° with respect to the **E** field of the linearly polarized incident laser light. In this case the scattered light distribution changes drastically as shown in Fig. 2. In Fig. 2(a) the direction of observation is chosen parallel to the **E** field, and in Fig. 2(b) perpendicular to the **E** field at the entrance. In both cases surprising node structures are observed, which differ only in the positions of the nodes. A node in one pattern corresponds to an antinode in the second pattern and vice versa. Figure 3 illustrates this schematically. A node structure similar to Fig. 2(a) has been reported by Lohmann *et al.*[3] for polymer molecules, oriented by an external electric field, in a solution.

At the entrance to the rod, the **E** field of the incoming wave is parallel to the z axis and 45° to the optic axis. The **E** vector can be represented as a superposition of two linearly polarized components of equal amplitude and phase, one component is parallel (E_1) and the other perpendicular (E_2) to the optic axis (Fig. 3, x-z plane at $d = 0$). Along the path d the birefringence of the Plexiglas rod induces a phase shift δ between these two components, which depends on d[4]:

$$\delta = \Delta n 2\pi d/\lambda, \tag{1}$$

where Δn is the relative difference of the principal refractive indices n_1 and n_2 ($\Delta n = |n_1 - n_2|$) and λ is the laser wavelength.

The phase shift δ leads to an elliptically polarized wave propagating in the Plexiglas rod. In other words the resultant electric field vector **E** follows an ellipse with respect to the x-z plane as illustrated in Fig. 3 for some characteristic path lengths d. The magnitude of the electric field components E_x and E_z, which represent the axis of the ellipse, depend on the phase shift δ.

For $d = 0$ one starts with $\delta = 0$ and with the **E** field of the incoming wave parallel to the z axis, i.e., $E_z = E$ and $E_x = 0$. In the rod this starting condition reappears for phase shifts $\delta = 2k\pi$; $k = 0,1,2,3...$. For phase shifts $\delta = (2k+1)\pi$; $k = 0,1,2,3...$ the resultant wave is linearly polarized too, but with the polarization vector parallel to the x axis, i.e., $E_z = 0$ and $E_x = E$. As a dipole does not radiate in the direction of its axis, it is possible to detect E_x and E_z separately by changing the direction of observation from the z to the x axis and vice versa. Consequently, in Fig. 2(a) nodes and antinodes and in Fig. 2(b) antinodes and nodes can be observed along the path of the rod. From one node to the next antinode the **E** vector changes by 90°

Fig. 1. Scattered light distribution of a laser beam propagating through a Plexiglas rod along the rod axis (y axis), (diameter = 10 mm, length: about 220-mm, 2-mW He–Ne laser). The incoming laser light is linearly polarized with the polarization vector (E field) **E** in the drawing plane (z axis). The optic axis of the rod is parallel to **E**, the emitted light is detected perpendicular to **E** and to the rod axis.

Fig. 2. Simultaneous registration of the scattered light emitted (a) in the direction of E (E field of the incoming plane wave) at the entrance (z axis) and (b) perpendicularly to E (x axis). Experimental conditions are as in Fig. 1 but with the optic axis of the Plexiglas rod oriented at 45° to E at the entrance. The simultaneous registration is obtained by a mirror.

and the phase shifts by π. For path lengths d with the corresponding phase shift

$$\delta = (2k + 1)\cdot\pi/2; \quad k = 0,1,2,3...,$$

the resultant wave is circularly polarized, i.e., $E_z = E_x = E/\sqrt{2}$.

For the Plexiglas rod the relative difference of the principal refractive indices can be evaluated from Eq. (1) with the measured node separation $d_0 = 34$ mm, the corresponding phase shift $\delta = 2\pi$, and the wavelength $\lambda = 633$ nm of the He–Ne laser. One obtains $\Delta n = 1.9\times 10^{-5}$ that is rather small in comparison with commonly used materials like calcite ($\Delta n = 3\times 10^{-3}$). This is the reason for the relative large node separation d_0 that allows the impressive demonstration experiment for birefringence and elliptical polarized light as in Fig. 2. In usual demonstrations described in textbooks the effect of retardation is only detected behind the plate through analyzing the polarization of light. With the Plexiglas rod, however, it is possible to observe the evaluation of the retardation and the respective elliptical polarized light in the material itself. This is a more instructive approach.

III. APPLICATIONS

Plexiglas also has some exceptional mechanical properties. It is very easy to cut and polish. Along with the above described optical properties, many applications that are very useful for teaching and technical purposes can be realized.

In addition to the demonstration of birefringence and elliptical polarized light it is possible to obtain retardation plates in a very simple way. These "home-made" optical retarders are easy to fabricate. One only has to cut off pieces with the desired "retardation length" $d(\lambda)$ and to polish the surfaces. For a quarter-wave plate $d(\lambda/4) = d_0/4$, and for a half-wave plate $d(\lambda/2) = d_0/2$. This corresponds to 8.5 and 17 mm, respectively, for the case of the rod used in Fig. 2.

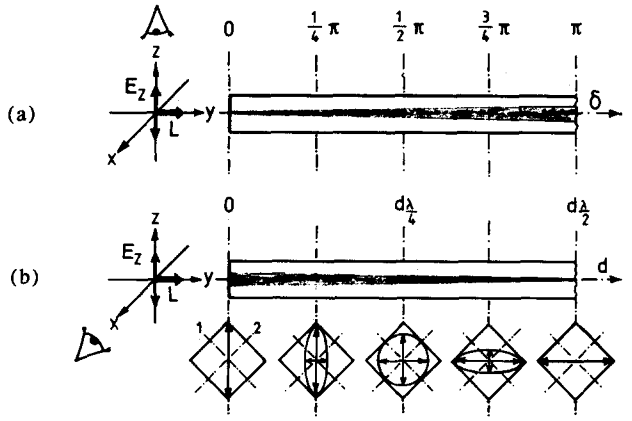

Fig. 3. Schematic drawing of the scattered light distribution in Fig. 2. For some characteristic phase differences δ the plane normal to the rod axis is drawn to illustrate the movement of the end point of the resultant E field vector in the respective plane. L: laser beam, propagating along the y axis (rod axis); E_z: z component of E; 1,2: optic axis and the axis perpendicular to it; δ: relative phase difference; d: path length in the rod.

These retardation plates are obviously very cheap, easy to make, and easy to handle. They are adjusted to a given wavelength with a high degree of precision and they are of "zero order." This could be important in some technical applications.

As the node distance depends on λ it is possible to use the Plexiglas rod as a spectrometer for laser light (the λ dependence of Δn is negligible in the visible region). Furthermore, the described method is well suited to measure small Δn values of materials that are transparent and show Rayleigh scattering in laser light.

[1] Manufacturer: Plexiglas: Röhm, Postfach 4242, D-6100 Darmstadt, Germany (type GS 222 or GS 233) or Rohm and Haas, Philadelphia, PA, USA; Lucite: E. I. du Pont de Nemours & Co., Wilmington, DE, USA; ask for cast-sheet Plexiglas or Lucite.

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[4] E. Hecht, *Optics* (Addison-Wesley, Reading, MA, 1987).